CONJUGATE CONDUCTION, CONVECTION AND RADIATION
PROBLEM IN WALLS CONTAINING CIRCULAR CELLS

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SUMMARY

The transient heat and mass transfer phenomenon through walls containing circular
cells is of great importance in simulation of thermal behavior in buildings. This pa-
per presents the numerical solution of the heat transfer problem, that is, conduction
in the brick and convection and radiation in the cell. Due to geometry complexity the
problem is solved using a boundary-fitted coordinate system. The results demonstrate
that, for the outer cell’s dimension studied, the heat flux is not significantly affec-
ted due the presence of the cavity, and that the radiation plays an important role.

INTRODUCTION

The analysis of the heat transfer through solid walls containing cavities is a challenging problem for
the fluid dynamicist. First, the solid part of the wall is itself a porous media where a conjugate heat
and mass transfer problem need to be solved. Coupled
to this problem one has the convection-radiation
problem inside the cavity. Associated to that, the
boundary conditions in a real problem are always
changing in time due to the transient nature of the
solar radiation, temperature and relative humidity of
the environment. To add a definite complicating
factor, it suffices to say that the real problem is
fully three-dimensional.

In spite of the complexity of the problem it is
possible to formulate some simplifying hypothesis such
that important engineering informations can still be
obtained.

Numerical and experimental studies dealing with
natural convection inside cavities without considering
conduction in the walls are abundant in the literature
and excellent reviews can be found [1,2], such that no
attempt will be made here of reviewing them.

Analysis of the conjugate problem, considering
conduction at the cavity walls and convection inside
the cavity, has also received attention but mainly for
square and rectangular geometries with geometrically
similar cavities [3,4,5]. The problem considering also
radiation at the cavity walls did not motivate too
much research work [6], and again they are restricted
to geometries whose boundaries follow the cartesian
coordinate system.

This paper presents a two dimensional
methodology with the analysis of a coupled problem,
conduction in the solid, convection-radiation in the
cavity, for arbitrary cavities. The methodology herein
presented is a powerful tool to simulate this coupled
problem for different geometries as well as to serve
as a basis for the solution of more complicated
mathematical models which may include 3D effects.

PROBLEM FORMULATION

The cellular wall in consideration is depicted
in figure 1a. For a large wall in the y and z direc-
tions, it is reasonable to assume no temperature chan-
ges in the z-direction as well as to assume that in
the y-direction one has a repetitive boundary condi-
tion. As a result the analysis is performed in the
cell shown in figure 1b. The additional simplifying

Figure 1. (a) Cellular wall (b) Cell

hypothese are as follow:
1. The humidity content is constant in the brick
such that only the heat transfer problem is
considered.
2. Brick and fluid properties are constant, except
the fluid density, responsible for the buoyancies
effects inside the circular cavity.
3. Boundary conditions are of constant temperature
at the vertical walls and insulated at the horizontal
ones. The latter condition will be again addressed.
4. At the solid fluid interface a continuous
temperature and heat flux profiles are assumed. In
addition it is assumed that the emissivity of the wall
is 1.0. This is not a severe assumption since normal
building bricks have emissivity in the order of 0.93.
This assumption allows to use the black body model for
the radiation exchange problem.

GOVERNING EQUATIONS

As mentioned, the governing equations need to be
solved in the solid domain and in the circular cavity.
With the listed assumptions these equations have the
following forms. For the solid part,
\[
\frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2} = 0
\]
and for the fluid inside the cavity,
\[
\frac{\partial (p\nu)}{\partial x} + \frac{\partial (p\nu)}{\partial y} = 0
\]
\[
\frac{\partial (p\nu)}{\partial x} + \frac{\partial (p\nu)}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left[ \mu \frac{\partial \nu}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \mu \frac{\partial \nu}{\partial y} \right] + \rho g
\]
\[
\frac{\partial (p\nu)}{\partial x} + \frac{\partial (p\nu)}{\partial y} = \frac{x}{d} \left[ \frac{\partial T}{\partial x} \right] + \frac{y}{c_p} \left[ \frac{\partial T}{\partial y} \right]
\]
It is important to note that the usual Boussinesq approximation is not being used. Instead, the density changes locally and is related to temperature and pressure through the state equation for ideal gases.

**BOUNDARY CONDITIONS**

For the solid wall, according to figure 1, one has
\[
T(0, y) = T_c
\]
\[
T(L, y) = T_H
\]
\[
\frac{\partial T}{\partial y} (x, 0) = 0
\]
\[
\frac{\partial T}{\partial y} (x, L) = 0
\]
The symmetry condition, originating eqns. (8) and (9) is an approximation since the natural convection creates different temperature profiles for the top and bottom of the circular cavity. This, through conduction in the solid, will also interfere in the temperature distribution at the top and bottom of the solid part. This difference will decrease as the ratio D/L decreases. In fact, for the situation tested (D/L = 0.5) the approximation is good. It is worth to note that the repetitive boundary condition could have been used without introducing any simplification. The insulated boundaries were used for the sake of simplicity only.

For the solid/fluid interface one has
\[
u|_O = 0
\]
\[
u|_0 = 0
\]
\[
T_s|_O = T_s|_0
\]
\[
-k_s \frac{\delta T}{\delta n}|_O = -k_s \frac{\delta T}{\delta n}|_0 + q_i
\]
where the symbols "O", "s" and "f" indicate the cavity wall, air and solid, respectively, and the second term in the right hand side of eq. (13) is given by
\[
q_i = \sum_{j=1}^{n} F_{ij} (T^4 - T^4)
\]
where \(q_i\) is the radiant heat flux exchanged by the element "i" with the "n" elements that can be "seen" by element "i". It is not a difficult matter to calculate the shape factor \(F_{ij}\) for this situation.

**NUMERICAL METHODOLOGY**

The problem is solved using a two-domain procedure, that is: the conduction equation, eq. (1), is solved using the heat flux from the convective problem added to the radiative heat flux given by eq. (14). The temperature from this solution serves as boundary condition for eqs. (2-6). The procedure is repeated until convergence.

Since the geometry is considered non-Cartesian, it is employed two boundary-fitted coordinate systems, as can be seen in figure 2. The one employed for the fluid equations is simply connected, and its transformation onto a rectangular domain retains as boundary conditions, in the four sides of the transformed plane, the existing boundary conditions in the physical plane. For the solid equation the domain is doubly-connected and for mapping again onto a rectangular domain it is necessary to use the cutting procedure. This results in a computational plane with repetitive boundary conditions along the cuts, as shown in figure 3. Or, in other words, this means to say that there is no need of specifying boundary conditions in the cuts. This, of course, is in accordance with the physical problem which does not provide boundary conditions in these regions. As can be seen in figure 3, the boundary conditions are all associated with the south and north frontiers of the computational domain.

![Figure 2. Computational grid for D/L = 0.5](image)

![Figure 3. Mapping of the solid domain](image)

The transformed equations are not presented here but the procedure to obtain them and details of the transformation can be found in [7]. The computational grid shown in figure 2 was generated using elliptic equations [8], with 20 x 20 volumes in the fluid domain and 80 x 20 in the solid part. It is to
observe that the grids fits well at the boundaries making it easier to transfer informations (boundary conditions) from one domain to the other.

The solution of the natural convection problem was realized using the computer code MACHED [9] with the appropriate simplifications. This code employs a co-located variable arrangement [10]. To handle the pressure-velocity coupling the SIMPLEC [11] method is employed, and the solution of the linear system of equations from the convective problem is obtained using MGS [12], and Gauss-Seidel for the conduction problem in the solid domain.

SOLUTION PROCEDURE

a) To save computer time during the iterative procedure the average temperature of \( T_H \) and \( T_C \) is initially assigned to all volumes in the domain.

b) Solve the equation in the solid wall assuming, in the first iteration, no heat flux at the solid fluid interface. Compute interface temperature using interpolation.

c) Solve for velocity and temperature for the fluid using as boundary condition the interface temperatures obtained in item b.

d) Compute the heat flux due to the convective flow.

e) With temperatures of item b it is calculated the flux by radiation using eq. (14).

f) The total heat flux, item d plus item e, is now used as boundary condition for the solid domain in item b, iterating until the prescribed convergence criterion is reached.

BASIC TEST CASES

When computing a numerical solution it is always desirable to have analytical solutions to test the numerical model. Of course, the problem one is dealing with precludes the determination of the analytical solution, but it is possible to analyse limiting cases. Using the full solution procedure, iterating from one domain to the other, density was kept constant and the air conductivity was made equal to the conductivity of the brick. The problem is then one-dimensional. It can be seen in figure 4 that the isotherms computed in each domain matches very well at the interface, reproducing the one-dimensional conduction problem. The maximum error of the numerical solution for this case, with \( T_H = 320 \, \text{K} \), \( T_C = 300 \, \text{K} \), \( \kappa_{\text{brick}} = 0.69 \, \text{W/mK} \), \( L = 0.1 \, \text{m} \) and \( D = L/2 \) is 4.3 %.

As a second basic test the coupled problem is solved using no radiation. The qualitative behaviour of the natural convection flow is in accordance with the solution of the natural convection inside a circular cavity with no conduction considered at the walls. The isotherms for this case is shown in figure 5.

NUMERICAL RESULTS AND DISCUSSIONS

With the main goal of better understanding the heat transfer phenomena involved, six different situations are simulated, as below.

1) Vacuum in the cavity with and without radiation.
2) Conduction in the air in the cavity with and without radiation.
3) Convection in the air in the cavity with and without radiation.

For each situation above it is varied the diameter of the cavity.

This problem is very rich in informations if all parameters are varied, forming the dimensionless groups which govern the phenomenon. In this paper this complete analysis is not realized and this is the reason why the results will be shown keeping the dimensions in the variables involved. The full analysis is under preparation and will appear in a coming publication.

\[ k_{eq} = \frac{h}{k_{\text{brick}}} \]

heat transfer considering the cavity

\[ k_{eq} = \frac{h}{k} \]

heat transfer considering the wall fully solid

where the denominator of the above equation is easily obtained exactly. Figure 6 shows the equivalent conductivity for the six different situations as a function of \( D \), using \( T_H = 320 \, \text{K} \), \( T_C = 300 \, \text{K} \), \( k_{\text{brick}} = 0.69 \, \text{W/mK} \) and \( L = 0.1 \, \text{m} \).

One important finding is the fact that the existence of the cavity, considering convection and radiation does not impair too much the heat transfer. In fact, the minimum value for \( k_{eq} \) encountered was 0.87, which means that the brick with the cavity exchanges 13% less heat than the wall fully solid. The results of figure 6 also demonstrate the
The main conclusions of this paper are twofold. First, it became clear that the analysis of complex problems involving complex domains can be done efficiently using a boundary-fitted grid. Several analyses can now be done for different shapes of the solid part and cavity.

Secondly, considering the physics of the phenomena, it was seen that, for the case tested, the existence of the cavity does not reduce significantly the heat transfer rates. The results also confirm that the radiation plays an important role in this type of problem, besides the low temperatures involved. However, as L and D decrease, the relative importance of the radiation and convection in the cavity also decreases. As in insulating materials, conduction tends to be the principal heat transfer mechanism in the air cavities. As already mentioned, in a coming paper, the conclusions will be generalized for the problem, considering the dimensionless groups involved.

REFERENCES


