SOLUTION OF A ONE-DIMENSIONAL THREE-PHASE FLOW IN HORIZONTAL WELLS USING A DRIFT-FLUX MODEL


Computational Fluid Dynamics Lab - SINMEC, Federal University of Santa Catarina (UFSC), Campus Universitário, Florianópolis-SC, Brasil, arthur@sinmec.ufsc.br, http://www.sinmec.ufsc.br

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Abstract. To design wells and its equipments along the oil fields, one has to solve the flow that occurs inside the oil reservoirs and its injecting and producing wells. This work presents a multiphase isothermal flow model to the solution for horizontal wells using a drift-flux model (water, oil and gas) along a one-dimensional domain. The problem is discretized with a Finite Volume Method and solved using Newton’s Method. As a drift-flux model is a homogeneous model, it uses the mixture momentum and continuity equations and it is also necessary to solve the gas and oil phase continuity equations in order to solve the three-phase flow. Lateral mass inflow, due to the flow from reservoir to well, is considered as a source/sink term in the one-dimensional continuity equations. Spatial and temporal interpolation schemes are of first order, mostly upwind schemes. Results are compared with a CMG-IMEX black-oil reservoir simulator. The algorithm is implemented using a C++ OOP programming language and all the derivatives of the Jacobian Matrix are calculated numerically, making the code more generic, allowing user to change property models and drift parameters according to the problem. The study of the coupling between well and reservoirs is a state-of-art research activity. Most of the important petroleum companies are developing proprietary softwares for modeling as much as possible the several phases of the oil production chain.
1 INTRODUCTION

For optimized petroleum exploitation, it is necessary to perform an intense sequence of studies and analysis. Reservoir simulation is a field that is in progressive growth since last decades and it’s been always increasing its complexity to better represent the physical phenomena that occurs during the process of oil extraction. Horizontal wells are today one of the alternatives to maximize production in several situations.

Another requirement is to consider multiphase flow along the reservoir and wellbore domains, for a better prediction of results and also different information like the total production of each phase. A complete reservoir simulation procedure requires the knowledge of pressure and saturation fields inside the reservoir subject to the boundary conditions that the injection and production wells provide. In order to obtain these boundary conditions, it is also necessary to determine the pressure and saturation fields along every well inside the reservoir (figure 1). Hence, these two problems need to be coupled in some manner. If this goal is achieved, one can use this tool to design and optimize well locations, well types, etc.

The solution of multiphase flow in horizontal wells can be done by solving the momentum and continuity equations for each phase, applying the proper interphase and wall friction models that may exist. This type of solution is know as multi-fluid models (Ishii and Hibiki, 2006). Yet, it can also be solved considering a homogeneous flow and then apply a model to represent the multiphase behavior, which are known as drift flux models. The main advantage of the latter approach is that it simplifies the problem so that fewer equations need solved and good results are still obtained.

Petroleum wells are usually represented as one-dimensional curves inside the reservoir with lateral mass inflow or outflow, as its dimensions are much smaller than the ones for the reservoir. In the presented work, we model a one-dimensional three-phase flow (water, oil and gas) corresponding to petroleum wells. The three-phase flow is modeled based on the approach proposed by Shi et al. (2003), but using the one-dimensional drift-flux model proposed by Hibiki and Ishii (2003).

2 MODEL FORMULATION

2.1 Drift-Flux Model

Drift-Flux models are homogeneous models that considers slip between phases allowing different velocities to be calculated for each phase. A complete explanation about those models and the calculation of its parameters can be found in Ishii and Hibiki (2006), where it presents the basic equations and formulas for the two-phase flow model. For the one-dimensional case, all properties in each control volume along the well should not vary in the cross-sectional directions. Therefore, it is necessary to integrate the properties along the cross-sectional area in order to obtain the one-dimensional equations. Given a property \( \phi \), its average over the cross-sectional area \( A \) is

\[
\langle \phi \rangle = \frac{1}{A} \int_A \phi \, dA
\]

(1)

the average operator \( \langle \ ) \ will be omitted from equations from now on to simplify notation. All the one-dimensional equations are derived from the integral along the cross-sectional area. Other properties should also be defined, such as volumetric fraction \( (\alpha) \), which represents the ratio between the volume occupied by the phase \( p \) over the total volume. Assuming that inside a control volume the properties do not vary, the volumetric fraction can also be calculated as the ratio of the cross-sectional area occupied by the phase over the total area. For a phase \( p \) we have
The sum of the volumetric fraction for each phase is
\[ \sum_p \alpha_p = 1. \] (3)

The drift-flux model is a two-phase formulation that involves the calculation of the velocity of each phase with a given constitutive equation that involves the mixture center-of-mass velocity and the drift velocity of the dispersed phase (Hibiki and Ishii, 2003). The drift velocity \( V_{gj} \) is the velocity of the dispersed phase relative to the volume center of the mixture
\[ V_{gj} \equiv v_g - j \] (4)
where \( v_g \) is the gas phase velocity and \( j \) is the total volumetric flux, calculated as
\[ j = \alpha_l v_l + \alpha_g v_g. \] (5)

the velocity \( v_p \) of a phase \( p \) is defined as the ratio between the volumetric flux of the phase over the cross-sectional area occupied by the same phase. Now it is possible to define the mean drift velocity, which allows the formulation of the constitutive equations to calculate the velocities of each phase. The mean drift velocity is given by
\[ \bar{V}_{gj} = V_{gj} + (C_0 - 1)j \] (6)
and $C_0$ is called the profile parameter that can assume different values according to the distribution profile of disperse phase. Expressions to calculate the velocities for each phase can also be derived. For the two-phase (gas-liquid) flow model, we have

$$v_g = v_m + \frac{\rho_l}{\rho_m} \bar{V}_{gj},$$
$$v_l = v_m - \frac{\alpha_g \rho_g}{\alpha_l \rho_m} \bar{V}_{gj}$$

and the total volumetric flux can be calculated as

$$j = v_m + \frac{\alpha_g (\rho_l - \rho_g)}{\rho_m} \bar{V}_{gj}$$

where $v_m$ is mixture velocity, defined by

$$v_m = \frac{\rho_g \alpha_g v_g + \rho_l \alpha_l v_l}{\rho_m}$$

To extend the two-phase drift-flux model to a three-phase flow, we now consider that the liquid phase is a mixture of two fluids (oil and water) and apply again the model for these phases with new drift-flux parameters (figure 2):

$$\bar{V}_{ow} = \bar{V}_{ow} + (C_0' - 1) j$$

and the oil and water velocities are calculated as

$$v_o = v_l + \frac{\rho_w}{\rho_l} \bar{V}_{ow},$$
$$v_w = v_l - \frac{\alpha_o \rho_o}{\alpha_w \rho_l} \bar{V}_{ow}$$

This extension of the two-phase multiphase model to a three-phase flow may not be as precise as a three-fluid model. Although, as pointed by Shi et al. (2003), this approach can produce reasonable results and the expected behaviour if the drift-flux parameters are well adjusted to the flow patterns that may exist along the well.

2.2 Wellbore Governing Equations

In the present model, four equations need to be solved in order to calculate all the necessary properties, three continuity equations and one momentum equation:

* **Mixture Continuity Equation**

$$\frac{\partial \rho_m}{\partial t} + \frac{\partial (\rho_m v_m)}{\partial s} = \left( \dot{m} \right)_{\text{total}}$$
Gas Phase Continuity Equation

\[
\frac{\partial (\alpha_g \rho_g)}{\partial t} + \frac{\partial (\alpha_g \rho_g v_m)}{\partial s} = \left( \frac{\dot{m}}{V} \right)_g - \frac{\partial}{\partial s} \left( \frac{\alpha_g \rho_g}{\rho_m} \bar{V}_{gj} \right)
\]  (13)

Oil Phase Continuity Equation

\[
\frac{\partial (\alpha_o \rho_o)}{\partial t} + \frac{\partial (\alpha_o \rho_o v_m)}{\partial s} = \left( \frac{\dot{m}}{V} \right)_o - \frac{\partial}{\partial s} \left( \frac{\alpha_o \rho_o}{\rho_m} \bar{V}_{ow} \right) + \frac{\partial}{\partial s} \left( \frac{\alpha_o \rho_o}{\alpha_l \rho_m} \bar{V}_{lj} \right)
\]  (14)

Mixture Momentum Equation

\[
\frac{\partial (\rho_m v_m)}{\partial t} + \frac{\partial (\rho_m v_m v_m)}{\partial s} = -\frac{\partial P}{\partial s} - \rho_m g \sin(\theta) - \frac{f}{2D} \rho_m v_m |v_m| - \frac{\partial}{\partial s} \left( \frac{\alpha_g \rho_g}{\alpha_l \rho_m} \bar{V}_{gj} \right)
\]  (15)

where the terms \((\dot{m}/V)_p\) correspond to the source/sink of each phase associated with the inflow or outflow of fluids between well and reservoir. The mixture equations are obtained through the sum of the equations for each phase. This problem is solved for pressure \((P)\), mixture velocity \((v_m)\), gas volumetric fraction \((\alpha_g)\) and oil volumetric fraction \((\alpha_o)\).

The friction factor \(f\) adopted in this work is the Churchill’s correlation:

\[
f = 8 \left( \frac{8}{Re} \right)^{12} + \frac{1}{(A + B)^{1.5}} \right)^{12}
\]  (16)

and

\[
A = \left( -2.457 \ln \left( \left( \frac{7}{Re} \right)^{0.9} + 0.27 \frac{\varepsilon}{D} \right) \right)^{16}
\]

\[
B = \left( \frac{37530}{Re} \right)^{16}
\]

where \(\varepsilon\) is the pipe’s rugosity, \(D\) the internal diameter and \(Re\) the Reynolds number, calculated as

\[
Re = \frac{\rho_m V D}{\mu_m}
\]  (17)

where \(V\) is a characteristic velocity of the flow \((v_m\) in this case).

Even though the expression above provides a way to calculate the friction factor for laminar and turbulent flow, it is only valid for single-phase flow and without lateral mass influx. Therefore, it’s necessary to find a better way of calculating the friction factor that better predicts the pressure drop for the given conditions. Some alternatives are proposed by Ouyang (1998). The calculation of the friction factor will not be discussed in this work.

2.3 Reservoir Governing Equation

The equations that represent the reservoir’s behavior, for a three-phase flow are the conservation of mass to each phase involved. Usually we use the formulation by saturation, however, this formulation presents computational problems when the gas component is fully contained in oil phase, i.e., \(S_g = 0\). For resolve this situation we use the mass fraction formulation to express the mass conservation for each phase (standard black-oil model) (Aziz and Settari, 1979).
2.3.1 Continuity Equations

This section will expose the mass conservation as a function of mass fractions of three phases (water, oil, and gas). The black-oil model considers the three components in the mixture, and three phases where the oil phase can be compound with two components, oil and gas. This equations are

Water Conservation

$$\frac{\partial}{\partial t} (\phi \rho^m Z^w) = \nabla \cdot \left( \frac{k^w \rho^w}{\mu^w} \tilde{K} \nabla \Phi^w \right) - \bar{m}^w,$$

(18)

Oil Conservation

$$\frac{\partial}{\partial t} (\phi \rho^m Z^o) = \nabla \cdot \left( X^{oo} \frac{k^o \rho^o}{\mu^o} \tilde{K} \nabla \Phi^o \right) - X^{oo} \bar{m}^w,$$

(19)

Gas Conservation

$$\frac{\partial}{\partial t} (\phi \rho^m Z^g) = \nabla \cdot \left( \frac{k^g \rho^g}{\mu^g} \tilde{K} \nabla \Phi^g \right) + (1 - X^{oo}) \frac{k^o \rho^o}{\mu^o} \tilde{K} \nabla \Phi^o - \bar{m}^g - (1 - X^{oo}) \bar{m}^o,$$

(20)

where $\rho^m$ is medium mass density defined to

$$\rho^m = \sum_{p} \rho^p S^p,$$

(21)

and $\phi$ is the porosity of the porous media. $Z^p$, $k^p$, $\rho^p$ and $\mu^p$ are the mass fraction, relative permeability, mass density and viscosity of the phases, respectively. $X^{cp}$ is the mass fraction of the componente $c$ in the phase $p$ and $\tilde{K}$ is the absolute permeability tensor.

The variable $\Phi^p$ is the potential of the phase $p$ and represent the relation between the oil pressure and the water and gas pressure. This relationship is the capillary pressure as

$$\Phi^o = p^o - \rho^o g z,$$

$$\Phi^w = p^o - \rho^{cow} - \rho^w g z,$$

(22)

$$\Phi^g = p^o - \rho^{cog} - \rho^g g z.$$

Finally $\bar{m}^p$ is the mass per volume entering or leaving the domain, to each phase. Adding to the system of equations has been the global mass conservation equation given by

$$Z^w + Z^o + Z^g = 1,$$

(23)

this form we have the total system of equations for reservoir simulator formed by equations (18) to (20) and eq. (23).

2.3.2 Discretization

The finite volume discretization of the equations is performed by integrating the control volume $P$ in space and time. Thus, equations (18), (19) and (20) turn into

$$\left[ (\phi \rho^m Z^w)^t + \Delta t - (\phi \rho^m Z^w)^t \right]_P \frac{\Delta V_P}{\Delta t} = \sum_{f=1}^{nf} [\lambda_f T_f (\Phi^w_{NB} - \Phi^w_P)] - \bar{m}^w,$$

(24)
\[
\left(\phi \rho_m Z^o\right)^{t+\Delta t} - \left(\phi \rho_m Z^o\right)^{t} \Delta V_P \frac{n_f}{\Delta t} = \sum_{f=1}^{n_f} \left[ X^{oo} \lambda_j^o T_f \left( \Phi^o_{NB} - \Phi^o_P \right) \right] - X^{oo} \dot{m}^o,
\]

(25)

\[
\left(\phi \rho_m Z^g\right)^{t+\Delta t} - \left(\phi \rho_m Z^g\right)^{t} \Delta V_P \frac{n_f}{\Delta t} = \sum_{f=1}^{n_f} \left[ \lambda^g_j T_f \left( \Phi^g_{NB} - \Phi^g_P \right) \right] + X^{go} \dot{m}^o - X^{go} \dot{m}^o,
\]

(26)

where \(f\) are the faces of the discrete control volume, \(\lambda^p_j\) is the mobility of phase \(p\) in the face \(f\) given by

\[
\lambda^p_j = \frac{\rho^p k^p}{\mu^w},
\]

(27)

and \(T_f\) is the transmissibility of face, where for the east face of control volume, on isotropic medium, is written as

\[
T_e = K \frac{\Delta y \Delta z}{\Delta x}.
\]

(28)

Finally, the \(\dot{m}^p\) is the mass flow rate that is entering or leaving the control volume by the well, which is determined from

\[
\dot{m}^o = \pm \lambda^o WI \left( P^o - P_{well} \right),
\]

\[
\dot{m}^w = \pm \lambda^w WI \left( P^w - P_{well} \right),
\]

\[
\dot{m}^g = \pm \lambda^g WI \left( P^g + P_{cog} - P_{well} \right),
\]

(29)

where \(WI\) is well index determined with the Peaceman’s wellmodel (Peaceman, 1983). This system of equations is composed of three unknowns, \(P^o\), \(Z^o\) and \(Z^w\) because \(Z^g\) can be determined by eq.(23).

3 NUMERICAL PROCEDURE

The discretized equations are solved using a Newton’s method, but reservoir equations and wellbore equations are solved separately. Therefore it is necessary to define a computational algorithm that couples both solutions and guarantee the convergence of the problem. The pressure along the well is a boundary condition for the solution of the reservoir flow and, once the pressure and saturations fields are calculated, it is possible to calculate the lateral mass inflow along the well. Therefore, the communication between reservoir and well is made through source/sink terms. So it is necessary to solve iteratively wellbore and reservoir equations until the fields are converged, always updating the boundary conditions of each domain. Figure 3 shows a simplified diagram of this procedure.

All the derivatives used to compute the jacobian matrix are calculated numerically, as follow

\[
\frac{\partial R}{\partial X} = \frac{R(X + \Delta X) - R(X)}{\Delta X}
\]

(30)

where \(R\) is one of the residual equations obtained after the discretization of the PDE’s and \(X\) one of the independent variables of the problem. This procedure is used for both reservoir and wellbore equations and provides a way of make the algorithm more general, allowing the inclusion of different property models in future studies, such as the ones proposed in Beggs (2003).
4 RESULTS

For the numerical tests, we created a synthetic reservoir with dimensions of $550m \times 550m \times 30m$ with the geometry given in figure 4. The reservoir is perforated with one vertical injector well and one horizontal producer well. It also has initially certain amounts of gas, oil and connate water. The simulation runs for 500 days and results are compared with the black-oil reservoir simulator IMEX-CMG and the simulation parameters are given on the appendix. Figures 5 and 6 show the total production of oil and gas and well as the flow rates of these phases, respectively. The drift-flux parameters inside the well were considered constant values and a no-slip condition for the oil-water model was imposed, so that oil and water have the same velocity.

During the initial part of the simulation, a great amount of gas is produced, chiefly during the first 100 days. After that, as the reservoir runs out of gas, the production of oil increases, but with a decreasing rate.

We did not found information about the drift-flux parameters and friction factor used on the CMG-IMEX well simulator. That could be one of the reasons why there’s a slight difference between the results of gas and oil production. Different drift-flux parameters and friction factor can result in a different pressure drop along the well, thus affecting the total production.

5 CONCLUSIONS

This paper presented a procedure for solving a three-phase flow along the well with a coupled framework with the reservoir. The model is based on a general algorithm that allows the
Figure 4: Scheme of the problem.

(a) Results for total gas production.  
(b) Results for total oil production.  
(c) Results for water cut.

Figure 5: Comparisons with CMG-IMEX.

(a) Gas flow rate.  
(b) Oil flow rate.

Figure 6: Comparisons of total rate with CMG-IMEX.
inclusion of different models for the coefficients in each equations, e.g., different density, viscosity and friction factor equations could be used. Also, the drift-flux parameters can be chosen according to each flow pattern that will occur inside the well, that depends on the volumetric fractions of each phase and its velocities (Provenzano, 2007). The design of an algorithm that does not depend on the fluid models and constitutive equations was one of the main goals of the presented work. Now it is possible to perform an indefinite sequence of studies of the correct parameters to use on each problem and obtaining results that better represent and predict the real situation of oil production.

As next steps, the inclusion of different drift-flux parameters according to each flow pattern, thus allowing the algorithm to handle most of the flow regimes. Also, the solution of the energy equation would allow the model to handle not only the horizontal part of the well, but also the vertical part (until the surface). The energy equation is necessary in the vertical part because the temperature gradients are much greater on that region, thus affecting the fluids properties.

REFERENCES

APPENDIX - PROPERTIES AND SIMULATION INFORMATION

Medium physical properties:

- Porosity ($\phi$): 0.2
- Rock Compressibility: $5.8015 \times 10^{-10} \frac{1}{Pa}$
- Reference Pressure Pore: $1 \times 10^5 Pa$
- Absolute permeability on the $x$ direction: $k_{xx} = 9.869^{-14} m^2$
- Absolute permeability on the $y$ direction: $k_{yy} = 9.869^{-14} m^2$
- Absolute permeability on the $z$ direction: $k_{zz} = 9.869^{-15} m^2$

Fluids physical properties:

Table 1: PVT tables.

- Reference Water Density ($\rho_{w,\text{ref}}$): 1000 kg/m$^3$
- Reference Oil Density ($\rho_{o,\text{ref}}$): 800 kg/m$^3$
- Reference Gas Density ($\rho_{g,\text{ref}}$): 1.1245 kg/m$^3$
- Water Reference Formation Volume Factor ($B_{w,\text{ref}}$): 1.01420
- Water Compressibility ($c_w$): 4.3511 $\times$ 10$^{-10}$ $\frac{1}{Pa}$
- Oil Compressibility ($c_o$): 1.4504 $\times$ 10$^{-9}$ $\frac{1}{Pa}$
- Reference Water Viscosity ($\mu_{w,\text{ref}}$): 10$^{-3}$ Pa $\cdot$ s
- Water Viscosityibility: 0.0 $\frac{Pa \cdot s}{Pa}$
- Oil Viscosityibility: 0.0 $\frac{Pa \cdot s}{Pa}$
- Reference Water Pressure: 10$^5$ Pa
Table 2: Solubility tables.

(a) Water-Oil solubility table.

<table>
<thead>
<tr>
<th>$S_w$ [fr]</th>
<th>$P_{cow}$ [Pa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.22</td>
<td>48263.299</td>
</tr>
<tr>
<td>0.3</td>
<td>27579.028</td>
</tr>
<tr>
<td>0.4</td>
<td>20684.271</td>
</tr>
<tr>
<td>0.5</td>
<td>17236.8925</td>
</tr>
<tr>
<td>0.6</td>
<td>13789.514</td>
</tr>
<tr>
<td>0.8</td>
<td>6894.757</td>
</tr>
<tr>
<td>0.9</td>
<td>3447.3785</td>
</tr>
<tr>
<td>1.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

(b) Gas-Liquid solubility table.

<table>
<thead>
<tr>
<th>$S_l$ [fr]</th>
<th>$P_{cog}$ [Pa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.22</td>
<td>26889.5523</td>
</tr>
<tr>
<td>0.3</td>
<td>24131.6495</td>
</tr>
<tr>
<td>0.4</td>
<td>20684.271</td>
</tr>
<tr>
<td>0.5</td>
<td>17236.8925</td>
</tr>
<tr>
<td>0.6</td>
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</tr>
<tr>
<td>0.7</td>
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</tr>
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<td>0.8</td>
<td>6894.757</td>
</tr>
<tr>
<td>0.9</td>
<td>3447.3785</td>
</tr>
<tr>
<td>0.96</td>
<td>1378.9514</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Initial parameters:

- Bubble Point Pressure: $10^8 Pa$
- Reference Depth: 30m
- Reference Pressure Depth: $7 \times 10^6 Pa$

Relative permeability curves:

- Water-Oil curves: Corey Correlation (1,1)
- Connate Water Saturation: 0.22
- Irreducible Oil WSaturation: 0
- Gas-liquid curves: Corey Correlation (1,1)
- Connate Gas Saturation: 0.0
- Irreducible Oil GSaturation: 0.0

Reservoir Simulation parameters:

- Timestep: 0.1 days
- Final Time: 500 days

Well physical parameters:

- Injection Vertical Well Bottom Hole Pressure: $3.86110 \times 10^7 Pa$
- Production Horizontal Well Bottom Hole Pressure ($P_{head}$): $2.75790 \times 10^6 Pa$
- Well Radius: 0.07m
- Well Rugosity ($\varepsilon$): $10^{-4} m$
Well Drift-Flux parameters:

- Gas-Liquid Drift Velocity $V_{gj}$: 0.05 m/s
- Gas-Liquid Profile Parameter $C_0$: 1.2
- Oil-Water Drift Velocity $V_{ow}$: 0.0 m/s
- Oil-Water Profile Parameter $C_0$: 1.0