Derivation of a Plasma-Actuator Model Utilizing Quiescent-Air PIV Data

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Abstract

The present work is concerned with development of a new model for the momentum transfer due to plasma-actuator in aerodynamic flow-control applications. A comparative analysis of different volume-force estimation strategies is provided, focusing particularly on the phenomenological and experiment-based approaches. The main objective of the present study is a comprehensive evaluation of these models aiming at derivation of a new model formulation of improved accuracy. All models are implemented into the OpenFOAM code. The models are validated within the RANS framework applied under the quiescent-air conditions by solving the Reynolds equations, closed by a near-wall second-moment closure model based on the homogeneous dissipation rate of the kinetic energy of turbulence. The computationally obtained velocity distribution induced by the wall-mounted plasma actuator is analyzed within the wall-jet flow region along with the experimental PIV data of identical actuator operating conditions. The results demonstrate the improved accuracy of the new empirical model as a first step towards a universal plasma-actuator model for flow-control simulations by means of CFD.

1 Background and Motivation

Plasma actuator for the aerodynamic flow control describes a DBD-based actuator (DBD: dielectric barrier discharge) generating an electric discharge between two parallel electrodes separated by an insulating dielectric material (barrier): a grounded electrode and the radio-frequency high-voltage one - the upper surface of the latter being coincident with the wall surface. Consequently an electric field originating from the exposed high-voltage electrode will ionize weakly the surrounding air (surface plasma generation) and induce a zero-mass wall-jet accelerating the flow in the immediate wall vicinity, Fig. 1(a). Applied to an existing boundary layer as sketched in Fig. 1(b), a delay of laminar-turbulent transition can be achieved [4] or flow separation can be suppressed [2]. Advantages and disadvantages of discharge based devices are discussed alongside other actuators for (active) flow control by Cattafesta and Sheplak [3]. An excellent review of the actuators’ working principle is provided by Moreau [11].

The determination of the magnitude and distribution of the force imparted from the plasma actuator to the external flow is of crucial importance for any advanced prediction of discharge-based flow-control scenarios by means of numerical simulations. The typical spatial and temporal scales of gas-discharge processes are four to eight orders of magnitude smaller than those of the resulting flow-control applications. To resolve this discrepancy for CFD (Computational Fluid Dynamics), several so-called phenomenological plasma-actuator models arose in recent years (see e.g. Jayaraman and Shyy [6]), each of which providing a temporally constant volume-force distribution \( f(x,y) \). Based on the necessarily strong simplification, the spatial...
distribution of these models typically result in rather artificial and non-physical shapes of the momentum-transfer domain. Therefore, the computational studies performed in the past only resulted in a moderate success, see e.g. Rizzetta and Visbal [13].

A promising alternative to provide an appropriate source term in the momentum equation within a CFD solution procedure, therefore, is the retroactive estimation of the volume force from experimental results. In the present work the PIV measurements in close proximity to dielectric-barrier discharge plasma actuators are conducted to achieve a velocity data base. This velocity information is then used for the force-estimation purpose and the development of an empirical model for the prediction of DBD plasma-actuator momentum transfer in numerical simulations. The major advantage of the proposed approach is the successful combination of the accurate spatial resolution of velocity-information based force-distribution determination with an efficient straightforward implementation of simple functions as used for phenomenological models.

3 Volume-Force Estimation Approaches

One major objective of the present study is a comparative analysis of advantages and drawbacks of different force estimation approaches. On the one hand any numerical simulation of discharge-based flow control necessarily involves strong simplification and partly non-physical assumptions to achieve phenomenological discharge-force models. On the other hand limitations of available measurement techniques require similar assumptions for experiment-based determination of the volume-force characteristics.

Both, phenomenological and experiment-based approaches as presented in this study are first briefly recapitulated in this section. Subsequently, the new empirical model is introduced. The processing flowchart of the present work is summarized in Fig. 3. All approaches considered presume the volume force to act quasi-steady, which is an appropriate approximation at least at aerodynamic scales. To assure best possible comparability all model calibrations were conducted such that the integral force magnitude

\[ F = \int_A f(x,y) \, dA \]  

was identical for all approaches and furthermore verified with force-balance data of Kriegseis et al. [8].

3.1 Phenomenological Models

Numerical simulations of plasma-actuator based flow-control commonly apply phenomenological models based on Shyy et al. [14] or Suzen et al. [15]. The basis of phenomenological models is the Coulomb-force equation

\[ \vec{f} = \rho_c e \vec{E} \]  

where charged particles of charge density \( \rho_c \) are accelerated in an external electric field \( E \).

For a given actuator geometry and operating voltage Shyy et al. [14] simplify the external electric field \( E \) to a triangle. The remaining unknown \( \rho_c \) is calculated using the first Maxwell’s equation

\[ \nabla \cdot \vec{E} = \frac{\rho_c}{\varepsilon_0} \]  

such that

\[ \vec{f} = \varepsilon_0 \vec{E} \cdot \nabla \cdot \vec{E} \]  

for the calculated distribution of \( E \). However, this approach is a strong oversimplification, since Eq. (4) does not distinguish between the external field for
Eq. (2) and the local field created by presence of charges in Eq. (3).

Suzen et al. [15] overcome the problem by dividing the electric potential $\Phi$ into an external and local component $\Phi = \phi + \phi_l$. The external field $E = \nabla \phi$ is determined by a Laplace equation for the external potential

$$
\nabla \cdot (\varepsilon \nabla \phi) = 0. \tag{5}
$$

The local field $\nabla \phi_l$ and the presence of charges are balanced by Eq. (3), which now includes the two unknowns $\phi$ and $\rho_c$. The assumption of equilibrium plasma allows the Boltzmann’s relation to be used, which in combination with the Debye shielding length $\lambda_d$ gives rise to

$$
\phi = -\frac{\rho_c \lambda^2}{\varepsilon_0}. \tag{6}
$$

Replacement of the local potential $\phi$ as defined by Eq. (6) leads to a Helmholtz equation for the charge density

$$
\nabla \cdot (\varepsilon \nabla \rho_c) = \frac{\rho_c}{\lambda_d^2}. \tag{7}
$$

In the present work, equations (5) and (7) are solved using COMSOL Multiphysics Partial Differential Equation (PDE) solvers. Thus, the required field strength $\nabla \phi$ and charge density $\rho_c$ were available for solving Eq. (2) for the volume force distribution $f(x,y)$.

### 3.2 Velocity-Information Based Model

The velocity-information based force-determination approaches rely on the two dimensional, steady Navier-Stokes equation (NSE) or vorticity equation, in accordance with the proposals of Wilke [16] or Albrecht et al. [1], respectively.

Wilke [16] assumed that the influence of the unknown pressure gradient is negligible such that the NSE reduces to

$$
\tau = \rho \left( u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \right) - \eta \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right). \tag{8}
$$

The force distribution according to Wilke’s method is sketched in Fig. 3 (middle-right).

Utilization of the vorticity equation eliminates the pressure gradient, but leaves the curl of the volume force, i.e. one equation with two unknowns. Albrecht et al. [1] assume the wall normal component of the
force to be at least one order of magnitude smaller than the streamwise one. Subsequent reintegration of the force gradient leads to

\[ f = -\rho \int_0^L \left[ \frac{\partial \omega}{\partial x} + \frac{\partial \omega}{\partial y} \cdot n \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) \right] dy. \quad (9) \]

Both experiment-based approaches are extensively described by Kriegseis [7].

### 3.3 New Empirical Model

The motivation for the development of an empirical (non-phenomenological) model is twofold. On the one hand, one wants to overcome the enormous efforts of conducting whole-field-technique measurements and to facilitate the consequent employment of the large database for CFD. In order words, one would like to replace it by a straightforward use of applicable equations. On the other hand, in contrast to the oversimplifying phenomenological models, the developed equations must represent the spatially accurate volume-force distribution of the data achieved by the experiment-based approaches [1, 16].

In the present work the spatial force distribution \( f(x, y) \) calculated according to Eq. (8) for quiescent-air PIV experiments was considered as underlying data base (see Fig. 3, middle-right). The corresponding integral force value for the electrical operating conditions is \( F = 25 \text{ mN/m} \) (cp. [7, 8]). The governing equations describing the magnitude of the force components in \( x \)-direction and \( y \)-direction are

\[ X(x) = (C_1 x + C_2 x^2) \exp(-x) \quad \text{and} \quad (10) \]
\[ Y(y) = (C_3 y + C_4 y^2) \exp(-C_5 y^2) \quad (11) \]

Scalar multiplication of expressions (10) and (11) leads to the desired functional description of the force distribution

\[ f(x, y) = C_6 X(x) Y(y), \quad (12) \]

which is displayed in Fig. 3 (bottom right).

Model equation (12) sets an empirical relationship between the force distribution and its spatial domain by calculating dependent variables \((C_1, C_2, \ldots, C_6)\) for Eqs. (10)-(12) based on a least-squares fit (see e.g. Press et al. [12]).

### 4 Computational Method

Complementary to the experimental work the flow field induced by the present plasma-actuator was computationally investigated by the RANS (Reynolds-Averaged Navier-Stokes) method. The unknown Reynolds stress components are computed by solving their model differential equations in line with the homogeneous-dissipation-based \( (\epsilon_h = \epsilon - 0.5\nu \partial^2 \kappa / (\partial x_i \partial x_i)) \), near-wall second-moment closure due Jakirić and Hanjalić [5]. Presently the model equation for the Reynolds stress tensor is solved in conjunction with the equation governing the homogeneous part of the inverse turbulent time scale \( \epsilon_h = \epsilon / k \). The latter equation is derived directly from the \( \epsilon \)-equation according to the procedure given in Maduta and Jakirić [10] and is completely equivalent to it. By doing so a number of important advantages pertinent to the homogeneous-dissipation concept could be retained: proper near-wall shape of the dissipation rate profile was obtained without introducing any additional term and the correct asymptotic behavior of the stress dissipation components by approaching the solid wall is fulfilled automatically without necessity for any wall geometry-related parameter. In addition, due to sake of comparative assessment the near-wall \( k-\epsilon \) model from Launder and Sharma [9] was also applied.

All computations were performed using the code OpenFOAM, an open source Computational Fluid Dynamics toolbox, utilizing a cell-center-based finite volume method on an unstructured numerical grid and employing the solution procedure based on the SIMPLE procedure for coupling the pressure and velocity fields. The convective transport in momentum equations was discretized by the 2nd order central differencing scheme implemented in the deferred-correction manner. 1st order upwind scheme was applied in the equations governing the turbulent quantities.

The dimensions of the flow area (measurement window) accommodating the experimentally determined velocity field, i.e. the resulting volume force was \( L_x \times L_y = 8.87 \times 4.38 \text{ mm}^2 \). This area was "experimentally resolved" by a grid comprising \( N_x \times N_y = 90 \times 46 \) cells. The corresponding numerical resolution is \( N_x \times N_y = 28 \times 36 \). The experimentally obtained force field database was linearly interpolated to the numerical grid. The dimensions of the experimentally considered box-like flow domain, confined by the solid walls, in the vertical \( x \)-\( y \) plane are \( 450 \times 345 \text{ mm}^2 \) (spanwise dimension is 325 mm).
The 2D flow domain was computed numerically, with the boundaries of the vertical \(x-y\) plane being set appropriately to avoid effects of the secondary flow currents (in the spanwise direction) to the volume force, as it was observed in the experimental work. It should be said, that consequent 3D computations (not shown here) accounting for the experimentally investigated flow domain in its entirety, justified the present 2D discretization. The computational grid was squeezed towards the solid walls to provide the dimensionless height of the wall-next computational cells being \(\Delta y^+ < 1\).

5 RESULTS AND DISCUSSION

Fig. 4 displays the comparison of the mean velocity profile of the plasma-actuator induced flow at location \(x = 10\) mm. The profile shape is typical of a wall-jet flow representing a non-equilibrium flow affected by a near-wall Reynolds stress anisotropy. This is also the main reason for the superiority of the Reynolds stress model results over the ‘anisotropy-blind’ \(k-\varepsilon\) model. Therefore, the following comparative discussion about the varying actuator-model approaches considers only the Reynolds stress model computations.

![Mean velocity profile](image)

**Figure 4:** Mean velocity profile \(u(y)\) at location \(x = 10\) mm - comparison of the results obtained by new plasma-actuator model used in conjunction with the \(k-\varepsilon\) [9] and Reynolds stress [5] models.

The wall-jet-like mean velocity profiles obtained by employing different actuator models are depicted at several downstream locations of the discharge domain in Fig. 5.

The direct comparison of the two experiment-based force estimation approaches confirms the validity of the respective independent assumptions. Eq. (8) neglects the pressure gradient, whereas Eq. (9) implies no simplifications concerning the pressure term. Both profiles agree well with one another for all streamwise locations.

The insufficient spatial accuracy of the phenomenological models inside the plasma region \((0 \leq x \leq 6\) mm) is obvious. Downstream of the discharge domain \((x \geq 6\) mm) the fully developed wall jet can be predicted by Shyy’s model, since the correct overall momentum is imparted to the flow. Although Suzen’s model results in the shape-accurate wall-jet profiles inside the discharge domain, the velocity magnitude in the fully-developed wall-jet region is underpredicted. This is assumed to happen due to the wall-normal distribution of the force density, which is too much concentrated to the immediate wall vicinity (for the Suzen’s model), whereas a constantly decreasing force density is characteristic for the Shyy’s model.

In contrast, application of the new empirical model results in an accurate prediction of the wall-jet profiles inside as well as downstream of the discharge domain. Good agreement of the wall-jet profiles inside the plasma demonstrate the improved spatial accuracy of the empirical model as compared to the phenomenological approaches. Consideration of the correct force magnitude \(F\) assures furthermore the accurate prediction of the maximum wall-jet velocity downstream of the momentum-transfer domain for \(x \geq 6\) mm.

6 CONCLUSIONS AND OUTLOOK

In the present work a new model formulation is proposed to simulate discharge-based flow manipulation. The major objective behind this new empirical approach is to provide a simple functional description of the force distribution in proximity of the plasma actuator, while assuring accurate spatial distribution of the force density.

The model sets an empirical relationship between the force distribution and its spatial domain. The flow field induced by the present model is computationally investigated by the RANS method solving the homogeneous-dissipation-based, near-wall second-moment closure model for the unknown Reynolds stress components [5]. The empirical model has been calibrated according to quiescent-air PIV experiments [7]. The present work shows that the numerical performance of the wall-jet flow inside the plasma region \((0 \leq x \leq 6\) mm) is extremely sensitive to different discharge-force models.

Depending on the simulated flow-control scenario, this sensitivity is a key factor defining success or failure of a CFD simulation. Since any counteraction of flow instabilities (e.g. Tollmien-Schlichting waves [4]) re-
requires a precisely adjusted force distribution to achieve successful flow-control results, an accurate prediction of this distribution is of utmost importance for a computational simulation of the flow behavior. The present model, therefore, is the attempt to contribute towards such an accurate force-distribution prediction, which is based on simple and straightforward applicable functional relationships.

As a first step, the new model predicts the wall-jet profile for the considered integral force value of $F = 25 \text{ mN/m}$. Upcoming work foresees the application of the new model to PIV result of varying actuator-force intensities, to derive appropriate equations of the model-coefficients as function of the underlying actuator force, i.e. $C_1 \ldots C_6 = C_1(F) \ldots C_6(F)$.

Based on this further improvement a universal application of the empirical modeling approach for numerical simulations of discharge-based flow-control applications will be targeted.

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**REFERENCES**


