Mixed Natural and Forced Convective Heat Transfer from a Horizontal Array of High Aspect Ratio Heated Isothermal Cylinders Placed in a Vertical Forced Flow

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ABSTRACT
Heat transfer from an array of identical horizontal cylinders with a rectangular cross-section and a relatively large height-to-width ratio has been numerically studied. There is a vertically upward forced flow over the cylinders which are all heated to the same uniform surface temperature. The flow conditions considered are such that mixed forced and natural convective flow exists. The flow has been assumed to be two-dimensional and the Boussinesq approximation has been adopted in dealing with the buoyancy forces. Attention has been restricted to the flow of air and results have therefore only been obtained for a Prandtl number of 0.74. Results for a relatively wide range of the other solution parameters have then been obtained. The flow conditions considered are such that laminar or turbulent flow can exist. Attention here has been directed at determining the effect of the flow parameters on the mean heat transfer rate from the cylinders.

1. INTRODUCTION
Flow over an array of cylinders that have a rectangular cross-section has been considered. The axes of the cylinders are horizontal with the longer sides of the rectangular cylinder cross-section being vertical. There is a vertically upward forced flow over the cylinders which have a uniform surface temperature that is higher than the temperature of the fluid flowing over the cylinder array. The flow situation is thus as shown in Fig. 1. The height, width and spacing between the cylinders are as defined in Fig. 1. The flow conditions considered are such that despite the presence of the forced flow over the cylinder array the buoyancy forces that arise due to the differences in temperature in the flow, in general, have an effect on the flow and thus on the heat transfer rate, i.e., mixed (or combined) forced and natural convective heat transfer exists. In the present study the heat transfer rate from the cylinders for a wide range of the parameters governing the flow has been numerically studied. Flow conditions in which laminar flow and in which turbulent flow occurs have both been considered.

Flows of the type here being considered occur in a number of situations involving the cooling of electrical and electronic equipment. Because such applications usually involve the flow of air over the cylinder array, attention has been restricted to air flow in the present study.

Earlier work on mixed convective heat transfer was concentrated mainly on flow over vertical flat plates. Typical of such studies are those of Gryzogoridis [1] and Merkin [2]. Some studies of mixed convective heat transfer from groups of plates have also been
Many studies of mixed convective heat transfer from horizontal cylinders have been undertaken including the studies of Badr [5, 6], Chang and Sa [7], Fand and Keswani [8], Gebhart and Pera [9], Gerlach et al. [10], Hatton et al. [11], Jackson and Yen [12], Jain and Lohar [13], Joshi and Sukhatme [14], Juma and Richardson [15], Kitamura et al. [16], Wong and Chen [17], Kitamura et al. [18-20], Merkin [21], Michaux-Leblond and Belorgey [22], Nakai and Okazaki [23], Noto et al. [24], Noto and Matsumoto [25], Oosthuizen and Madan [26], Sharma and Sukhatme [28], Shi et al. [29], Lee et al. [30], and Sparrow and Lee [31]. Some studies of mixed convective heat transfer from square horizontal cylinders have also been undertaken, e.g., see Bhattacharyya [32], Sharma and Eswaran [33], and Singh [34]. Mixed convective heat transfer from rectangular cylinders has received little attention and this was, in part, the reason for the present study.

2. Solution Procedure

The flow has been assumed to be steady and two-dimensional, i.e. the length of the cylinders in the horizontal axial direction has been assumed to be large compared to the width and height of the cylinders and the effect of the changes in the flow near the ends of the cylinders on the mean flow has thus been assumed to be negligible. The surfaces of the cylinders have been assumed to be maintained at the same uniform temperature, i.e. the cylinders have been assumed to be isothermal. The Boussinesq approximation has been adopted, i.e. the changes in fluid properties with temperature have been neglected except for the changes in density with temperature which has been treated by assuming a linear variation of density with temperature. When these assumptions are adopted the mean Nusselt number, $Nu$, is dependent on the following parameters:

$$ Nu = \text{function}(Re, Gr, Pr, W, S) $$

Here $Nu$ is the Nusselt number based on the height of the cylinders, $h$, $Gr$ is the Grashof number based on $h$, $Ra$ is the Rayleigh number based on $h$, $Pr$ is the Prandtl number, $W$ is the dimensionless width of the cylinders, and $S$ is the dimensionless spacing of the cylinders. As mentioned above, attention has been restricted to the flow of air and $Pr$ has therefore been assumed to be constant and equal to 0.74. The remaining variables are therefore defined as follows:

$$ Nu = \frac{\bar{q}h}{kA(T_a - T_s)}, \quad Re = \frac{Uh}{v}, $$

$$ Gr = \frac{\beta gh^4(T_a - T_s)}{v^2}, \quad W = \frac{w}{h} $$

$$ S = \frac{s}{h} $$

The results have been obtained by using the commercial CFD code Fluent®. The range of parameters considered is such that both laminar and turbulent flow can exist and the basic k-epsilon turbulence model with full account of buoyancy force effects accounted for has been used in obtaining the solution.

The flow over each cylinder in the array has been assumed to be the same and the flow about the vertical centre-lines through the cylinders and about the vertical centre-line between the cylinders has therefore been assumed to be symmetrical, i.e., the solution domain shown in Fig. 2 has been adopted.

Extensive grid- and convergence criterion independence testing was undertaken. This indicated that the mean heat transfer results presented here are to within 1% independent of the number of grid points and of the convergence-criterion used.
3. RESULTS

As discussed in the previous section the parameters on which the solution depends are $Re$, the Reynolds number based on $h$, $Gr$, the Grashof number based on $h$, $W$, the dimensionless width of the cylinders, and $S$ the dimensionless spacing of the cylinders in the array. Results have been obtained for $Re$ values between approximately 10 and $10^7$, for $Gr$ values between approximately $10^6$ and $10^{10}$, and for $W$ values of 0.125 and 0.25. Results have been obtained for $S$ values between 0.25 and 0.75 as well as for the case when $S$ is so large that the flows over each of the cylinders in the array does not influence the flow over adjacent cylinders and the flow over the cylinders is therefore the same as that over a single isolated cylinder.

The basic form of the results obtained is illustrated by the results given in Figs. 3 and 4. These figures show the variation of the mean Nusselt number with Reynolds number for various values of the Grashof number for a cylinder with a dimensionless width, $W$, of 0.25. The results given in Fig. 3 are for the case where the spacing between the cylinders is large and the flow over each cylinder is effectively the same as that over a single isolated cylinder while the results given in Fig. 4 are for a cylinder spacing, $S$, of 0.25.

The results given in Figs. 3 and 4 illustrate the relatively strong influence that the cylinder spacing has on the mean heat transfer rate. These results also illustrate how, at low Reynolds numbers, the Nusselt number is essentially independent of Reynolds number, i.e., that under these conditions effectively pure natural convection exists, while at high Reynolds numbers the Nusselt number is essentially independent of Grashof number, i.e., that under these conditions effectively pure forced convection exists. The variations of Reynolds numbers with Rayleigh number that define when purely forced convection and when purely natural convection exist for the conditions covered by the results in Figs. 3 and 4 are shown in Fig. 5.

The results given in Figs. 3 and 4 are for a cylinder with a dimensionless width, $W$, of 0.25. To illustrate the effect of $W$ on the form of the results Nusselt number variations for $W$ values of 0.125 and 0.75 are
shown in Figs. 6 and 7. These figures show the Variations of Nusselt number with Reynolds number for various values of the dimensionless cylinder spacing, $S$. It will be seen that the form of the Nusselt number variation is significantly different for the two values of $W$ considered. This difference is mainly because at the lower $W$ value considered the contribution of the heat transfer from the horizontal top and bottom surfaces of the cylinder is relatively small compared to that from the vertical side surfaces of the cylinder because of the relatively small surface areas of the horizontal side surfaces compared to the total cylinder surface area. At the larger $W$ value considered the contribution of the heat transfer from the horizontal top and bottom surfaces of the cylinder is much closer to that from the vertical side surfaces of the cylinder.

Typical variations of the Nusselt number with Reynolds number for $W = 0.75$ and $S = 0.5$ for two Grashof number values are shown in Fig. 8. If the conditions under which the flow can be assumed to be purely forced convective that are indicated by the results given in Fig. 8 are compared with the results given in Fig. 5 which were for $W = 0.25$ it will be seen that at the conditions under which the flow can be considered purely forced convective are similar for the two $W$ values.

**CONCLUSIONS**

The results of the present study indicate that:

1. The dimensionless cylinder spacing has a significant effect on the mean Nusselt number at the intermediate Reynolds number values considered.
2. Because of the changes with dimensionless cylinder width in ratio of the mean heat transfer rate from the horizontal top and bottom surfaces to the total mean heat transfer rate from the entire cylinder surface the nature of the effect of dimensionless cylinder spacing on the mean Nusselt number for the cylinder is significantly dependent on the dimensionless cylinder width.
3. The conditions under which purely forced convective flow can be assumed to exist are significantly dependent on the dimensionless cylinder spacing at the lower Grashof number values considered.
REFERENCES


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