Laminar-Turbulent Transition Prediction in a Reynolds-Averaged Navier-Stokes Solver

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ABSTRACT
A two-dimensional Reynolds-Averaged Navier-Stokes (RANS) flow solver is extended to incorporate an iterative laminar-turbulent transition prediction framework. The Arnal-Habiballah-Delcourt (AHD) criterion is used to predict the natural transition locations due to Tollmien-Schlichting instabilities. The transition to turbulent flow is modeled using the Spalart-Allmaras (SA) turbulence model, making use of the laminar-turbulent trip functions therein. The boundary-layer properties are obtained directly from the Navier-Stokes (NS) flow solution. This work presents a comparison of three methods that may be used to define the boundary-layer edge directly from the NS solution, followed by a grid convergence study and verification of the integrated boundary-layer properties. A robust iterative framework for transition prediction in a RANS flow solver is also presented. The predictive capability of the new framework (making use of the AHD criterion and the SA model) is then demonstrated and validated by comparison to experimental and computational airfoil transition data.

1 INTRODUCTION AND MOTIVATION
The current push for environmentally-friendly and sustainable aviation requires serious efforts to mitigate the escalating effects of such technology on climate change and natural resources. Airframe design considerations aimed at delaying the transition from laminar to turbulent flow (via passive and/or active flow control) have the potential to significantly reduce aircraft drag. Hence, the optimal aerodynamic shape of an airfoil should incorporate and exploit laminar-turbulent transition. In this work, a two-dimensional Reynolds-Averaged Navier-Stokes (RANS) flow solver is extended to incorporate an iterative laminar-turbulent transition prediction framework. The transition prediction approach presented is targeted for use in an aerodynamic shape optimization framework [1, 2], with subsequent extensions possible to three dimensions, incorporating crossflow instabilities.

The challenges in reliably predicting laminar-turbulent transition continue to limit our ability to predict many aerodynamic flows with accuracy [3]. Researchers continue to develop transition criteria with varying complexity and fidelity. For the subsonic and transonic flight regimes, there are several prediction methodologies compatible with RANS flow solvers. As categorized by Arnal [4], the three main engineering approaches are to (i) employ the eN-method [5, 6, 7, 8, 9, 10], (ii) solve additional transport equations [11, 12, 13], or (iii) employ simpler transition correlations [9, 14, 15, 16]. In the same review by Arnal [4], the level of fidelity, advantages, and disadvantages of the various approaches are compared.

The simpler correlation approach, specifically the Arnal-Habiballah-Delcourt criterion [15], is the method of choice in this work, as it presents a good compromise between accuracy, robustness and efficiency that is well suited to aerodynamic shape optimization. Although this approach is non-local (as discussed in Section 2.1), it avoids the extra computational effort required to solve additional partial differential equations. Correlations for crossflow instabilities, such as the C1 criterion, have been successfully combined with the AHD criterion [9, 16], in turn allowing for extensions to three dimensions. In addition, the proposed framework for transition prediction has been designed in a modular fashion, facilitating the use of higher fidelity criteria (such as the eN-method). Research in the area of aerodynamic shape optimiza-
tion with laminar-turbulent transition is sparse. With the exception of a few more recent works employing RANS flow solvers [7, 8, 17, 18], the majority of research has employed boundary-layer (BL) codes through inviscid-viscous coupling. The future objective of this work is to advance the state-of-the-art in aerodynamic shape optimization, by incorporating and exploiting the laminar-turbulent transition methodology presented in the next section.

2 Numerical Methodology

This section provides a brief overview of the numerical methodology used in the proposed transition prediction framework.

The RANS equations are solved in two dimensions using a second-order Newton-Krylov finite-difference flow solver developed by Nemec and Zingg [1, 2]. The linear system that arises at each Newton iteration is solved efficiently using the preconditioned generalized minimum residual method. Global convergence of the Newton method is made possible by an approximate factorization startup algorithm. Numerical dissipation is added by either the scalar dissipation scheme of Jameson et al. [19] or the matrix dissipation scheme of Swanson and Turkel [20]. The use of matrix dissipation is recommended for the present purpose in Section 3.2.1.

The solver (named Optima2D) makes use of the one-equation Spalart-Allmaras (SA) turbulence model, including the laminar-turbulent trip term and the \( f_1 \) and \( f_2 \) trip functions. This turbulence model is fully detailed in [21]. The SA model is not itself capable of predicting transition. However, when provided with the transition locations the SA model is designed to ramp-up the eddy-viscosity in a smooth manner.

The remaining constituents of the proposed framework include: the determination of the BL thickness and properties, the calculation and evaluation of the AHD transition criterion, and the implementation of a robust iterative procedure for transition prediction in the RANS solver.

2.1 Calculation of the Boundary Layer Edge and Properties

Many transition prediction methods for wall-bounded flows make use of BL properties. By definition, the various BL properties are non-local, since they require the integration of flow quantities from the wall to the BL edge. For example, the AHD criterion requires the calculation of the displacement thickness (\( \delta^* \)), momentum thickness (\( \theta \)), shape-factor (\( H \)), and Pohlhausen number (\( A \)), all of which require the calculation of the BL thickness (\( \delta \)).

For arbitrary pressure distributions the edge velocity, \( U_e \), is not known \textit{a priori}, since \( U_e(x) \neq U_w \), in general. Hence, we must somehow define the BL edge based on the RANS flow solution. One approach is to couple the RANS solver to a BL solver that provides a BL thickness as part of the solution process [5, 6, 7]. The benefit of using a BL solver is in the accurate calculation of the BL thickness while allowing for coarser grids to be used in the RANS solver [6]. The disadvantages of the coupling approach include: (i) redundancy in solving both the NS and BL equations, (ii) restriction to fully-attached and mildly separated flows, (iii) implementation and convergence issues in coupling the two solvers, and (iv) complexities when moving to compressible flows, wings of finite span, and parallel implementations. For these reasons, this work makes use of the available RANS solution, altogether avoiding the use of a BL solver.

Work completed thus far has involved the implementation of three BL edge-finding methods. A comparison and assessment of their accuracy may be found in Section 3.1. The methods are briefly summarized as follows:

**Compressible Bernoulli Equation:** Following Nebel et al. [22], the first method makes use of the local wall pressure, \( p_w \), to approximate the edge velocity via the Bernoulli equation for compressible flows,

\[
U_e = \sqrt{U_w^2 - \frac{2\gamma}{\gamma-1} \frac{p_w}{\rho_w} \left( \frac{p_w}{p_w} \right)^{\frac{\gamma-1}{\gamma}} - 1},
\]

The boundary layer thickness for the given streamwise station, is then searched in the off-wall direction for the point \( \delta=y \) at which \( U=0.99U_e \), where \( y \) is used here to denote the normal off-wall distance.

**Baldwin-Lomax Diagnostic Function:** Following Stock and Haase [23] and Nebel et al. [22], the second method makes use of a so-called diagnostic function derived from the Baldwin-Lomax turbulence model. The diagnostic function,

\[
F = y^b \left[ \frac{dU}{dy} \right]^a,
\]

is first computed and its maximum value in the off-wall direction is searched. The boundary layer thickness is then computed as \( \delta = \varepsilon \cdot y_{\text{max}} \), where \( y_{\text{max}} = y \) at which \( F = F_{\text{max}} \). The values of the constants have been deter-
mined through numerical and experimental investigation to be $a_i=3.9$, $b_i=1.0$, and $\varepsilon_i=1.294$ for laminar boundary layers, and $a_i=1.0$, $b_i=1.0$, and $\varepsilon_i=1.936$ for turbulent boundary layers [22].

**Vorticity and Shear-Stress Method:** Following Cliquet and Arnal [15], the third method makes use of the local vorticity, $\Omega$, and an approximation to the total shear stress, $\tau_{tot}$. The total shear stress is defined as $\tau_{tot}=\tau_l+\tau_t$, where $\tau_l$ and $\tau_t$ can be expressed in the following manner [15]:

$$\tau_l = \mu_l |\Omega| \quad \text{and} \quad \tau_t = \mu_t |\Omega| . \quad (3)$$

The boundary layer thicknesses based on vorticity and shear stress are then searched in the off-wall direction, such that

$$\delta_\Omega = y \quad \text{at which} \quad |\Omega| = \varepsilon_\Omega \cdot |\Omega|_{\text{max}}, \quad (4)$$

and

$$\delta_t = y \quad \text{at which} \quad |\tau_{tot}| = \varepsilon_t \cdot |\tau_{tot}|_{\text{max}}, \quad (5)$$

where $\varepsilon_\Omega=0.001$ and $\varepsilon_t=0.015$. Finally, the boundary layer thickness is taken as the minimum, such that $\delta = \min(\delta_\Omega, \delta_t)$.

In Section 3, an investigation into the accuracy of the integrated BL properties is considered through a detailed grid convergence study. With the BL edge defined (facilitating the calculation of the BL properties) the next step is to consider the evaluation of a transition criterion.

### 2.2 AHD Transition Criterion

The natural transition locations (due to Tollmien-Schlichting instabilities) are predicted using the new compressible form of the Arnal-Habiballah-Delcourt (AHD) criterion [9, 15, 16, 24]. The AHD criterion is designed for low to moderate freestream turbulence intensities ($T_t \leq 0.1\%$), as typically encountered in external aerodynamic cruise conditions for transport aircraft [15].

Beginning at the stagnation point, we march toward the trailing edge of the airfoil, treating the upper and lower surfaces independently. Our first task is to find the streamwise location of the neutral stability point, $s_{CR}$. Upstream of the neutral stability point it is assumed (from linear stability theory) that all small disturbances over all frequencies remain stable and damp out. The neutral stability point is found using the critical Reynolds number, calculated as a function of the incompressible shape factor, $H_i$, as

$$Re_{\theta CR} = \exp \left[ \frac{E}{H_i} - F \right], \quad (6)$$

such that $s_{CR}$ is the first point at which, locally, $Re_\theta = Re_{\theta CR}$. The constants E and F may be found in the Appendix. Note that $Re_{\theta CR}$ typically decreases in the streamwise direction and is greater than $Re_\theta$ upstream of the critical point.

The next step is to find the streamwise location of the laminar-turbulent transition point, $s_{TR}$. Note that the transition criterion is computed and checked only at points downstream of the neutral stability point. The basic idea of the AHD criterion is to use the Falkner-Skan self-similar solutions to represent the laminar boundary layer profiles, which are characterized by the local Pohlhausen number [15]. Following the work of Granville, the necessary relationships are extended from self-similar boundary layers to actual flows by replacing $\Lambda_2$ with its mean value as follows [15]:

$$\Lambda_2 = \frac{\theta^2}{L} \frac{dU_e}{v} \text{d}s \quad \Rightarrow \quad \overline{\Lambda_2} = \frac{1}{s-s_{CR}} \int_{s_{CR}}^{s} \Lambda_2 \text{d}s . \quad (7)$$

Arnal has proposed the following expression for the transitional Reynolds number, $Re_{\theta TR}$:

$$Re_{\theta TR} = Re_{\theta CR} + A \exp(B \Lambda_2) \left[ \ln(C-T_u) - D \Lambda_2 \right] , \quad (8)$$

where $T_u$ is the freestream turbulence level, and the constants A, B, C, and D may be found in the Appendix. The transition point is then taken as the first point at which, locally, $Re_\theta = Re_{\theta TR}$. Note that $Re_{\theta TR}$ typically decreases in the streamwise direction and is greater than $Re_\theta$ upstream of the transition point.

In the new compressible form of the AHD criterion (found in [24]), the constants A through F in Equations (6) and (8) are computed as a function of the freestream Mach number, $M_{\infty}$. The details of these polynomial expressions may be found in the Appendix.

### 2.3 RANS Implementation

Automatic transition prediction is achieved through an iterative process. An initial guess of the transition locations (top and bottom surfaces) is made far enough downstream (typically 70%-80% chord) to allow for a sufficient laminar boundary-layer region for transition prediction, but not so far downstream as to cause convergence issues in the flow solver. Upon flow solver
convergence, the transition prediction module is invoked to process the RANS solution. The initial guess is then moved upstream toward the newly predicted transition points in an under-relaxed fashion, such that

\[ X_{TR_{NEW}} = X_{TR_{OLD}} - \omega (X_{TR_{OLD}} - X_{TR_{PREDICT}}) \quad (9) \]

where \( \omega \) is the under-relaxation factor, and \( X = \frac{x}{c} \) is the normalized chord position. From numerical experimentation, an aggressive under-relaxation factor of \( \omega = 0.9 \) was found to be a good compromise between efficiency and robustness.

If a transition point is not successfully predicted upstream of the initial guess, the transition location is moved downstream and the flow solver restarted. If laminar flow separation is detected, then the separation point is taken as an approximation to the transition point and the flow solver restarted. In this fashion, convergence onto a transition point is possible. A robust logic has been determined through extensive numerical experimentation and code verification to handle the various outcomes of the transition prediction module.

For the various airfoils and flight conditions investigated thus far, it was found that the iterative transition prediction procedure requires approximately three to four times the computational cost of a single fully-turbulent flow solve.

## 3 RESULTS AND DISCUSSION

### 3.1 Boundary Layer Edge

In order to verify the three BL edge-finding methods (described in Section 2.1), numerical flow solutions using fixed transition locations were computed using Optima2D. The results from Optima2D are compared to numerical results obtained from XFOIL, developed by Drela [25]. XFOIL is a two-dimensional, inviscid solver which couples an inviscid code to a viscous BL code. The inviscid formulation in XFOIL is a linear vorticity-streamfunction panel method. The viscous flow in the BL and wake is modeled with a two-equation lagged dissipation integral BL method [25].

Flow solutions were computed on the NACA-0012 airfoil with a sharp trailing-edge at \( Re = 1 \times 10^6 \), \( M = 0.20 \) and zero incidence, using a C-grid with 321x384 nodes. Transition was fixed at 50% chord on both the top and bottom surfaces of the airfoil. A comparison of the three edge-finding methods in Optima2D is presented in Figure 1, along with the edge velocity obtained using XFOIL. The comparison verifies the ability of the various methods to define the BL edge, without the use of a BL solver. Good agreement is observed between the methods in Optima2D and XFOIL in both the laminar and turbulent regions.

### 3.2 Boundary Layer Properties

The accuracy of the integrated boundary-layer properties is assessed through a grid convergence study and by comparison to numerical BL properties obtained from XFOIL. Figure 2 presents a grid convergence study of the boundary-layer displacement thickness (top) and momentum thickness (bottom), for the same NACA-0012 test case as above. The grid-convergence results shown here were obtained using the vorticity...
and shear-stress edge-finding method. The numbers in brackets provide the approximate number of nodes in the laminar BL.

At first glance it appears as though $\delta^*$ and $\theta$ show reasonably good agreement amongst all the grids considered. However, the differences in their accuracy are made more evident by comparing their ratio, that is, the shape factor, $H = \frac{\delta^*}{\theta}$. Figure 3 shows the shape factor distributions for the same grid convergence study and demonstrates the importance of sufficient grid resolution in the BL. These results also demonstrate that (with reasonable grid sizes) sufficient accuracy of the boundary-layer properties may be obtained directly from the NS solution.

### 3.2.1 Some Remarks on Numerical Dissipation

All results presented in this paper (with the exception of Figure 4) have been obtained using a matrix dissipation scheme. Investigations into the accuracy of the BL properties revealed that the scalar dissipation scheme was overly dissipative. A clear improvement in the accuracy of the BL properties was observed when using the matrix dissipation scheme. Similar numerical dissipation effects on the BL properties have been reported by Mayda [10]. The improvement can be seen in Figure 4, which compares the shape factor obtained by scalar and matrix dissipation for the 321×256 grid, for the same NACA-0012 test case.

### 3.3 Transition Prediction

#### 3.3.1 Verification

The predictive capability of the transition prediction framework is first verified by comparison to numerical results generated using XFOIL on the NACA-0012 airfoil. The transition prediction in XFOIL makes use of a simplified $e^N$-envelope method developed by Drela [25].

In Figure 5 transition prediction results are shown for a 449×385 C-grid around a NACA-0012 airfoil at $Re = 10 \times 10^6$, $M = 0.2$, and a freestream turbulence intensity ($T_u$) of 0.1% (corresponding to an N-factor of 8 for XFOIL). The Optima2D (O2D) points represent the final converged transition locations using the AHD criterion and an initial guess at 80% chord for both the upper and lower surface transition points. We can certainly see that XFOIL and Optima2D produce similar trends in the transition point variation with angle of attack (AoA); the upper surface transition points move forward with AoA, and vice versa on the lower surface. It is also observed that the AHD criterion predicts transition upstream of XFOIL. Similar results have been found by Cliquet and Arnal [15] and Streit et al. [9].

Cliquet and Arnal have recommended shifting the AHD transition points based on an extrapolation of the laminar $Re_{\theta}$ on the order of 10% of $Re_{\theta_T}$ [15]. This shift has been implemented as an available option in the proposed framework. However, considering recent numerical comparisons by Campbell et al. [26], a shift in the AHD results (in this case to match XFOIL results) is difficult to justify. Campbell et al. found that MSES (and hence XFOIL) consistently predicts transition downstream (5 to 15% chord) of well-established and higher fidelity linear stability codes (LASTRAC and LILO) over a range of geometries and flight conditions. As such, the AHD transition results are not
shifted for any of the transition results presented in this paper. In some cases, the shift may be justified, particularly when attempting to match AHD results to experimental data for a given freestream turbulence intensity (as performed in [9] and [15]).

### 3.3.2 Validation

Validation of the transition prediction framework and the AHD criterion has been carried out by comparison to available experimental transition data for the NLF-0416 airfoil, developed by Somers [27]. The experimental results were obtained in the Langley Low Turbulence Pressure Tunnel (LTPT) using microphoned pressure taps [27]. As such, the resolution of the experiments corresponds to the physical spacing of the microphoned taps along the chord of the airfoil.

Presented herein are the test case results for a $449 \times 385$ C-grid around the NLF-0416 airfoil at $Re = 4 \times 10^6$, $M = 0.2$, and $T_u = 0.1\%$ (and $N = 8$ for XFOIL). The transition points predicted by both Optima2D (using the AHD criterion) and XFOIL (using the simplified $e^N$-envelope method) are shown in Figure 6, along with the wind tunnel experimental data. The results of this test case show that the predictive capabilities of Optima2D match closely with the published experimental results over a range of lift coefficients.

Figure 7 presents the drag polar for the NLF-0416 airfoil using both Optima2D and XFOIL. Good agreement is observed between the experimental results and the drag polars computed using both Optima2D and XFOIL.

In Somers’ report [27], the freestream turbulence intensity, $T_u$, was unfortunately not published for the NLF-0416 experiments. As such, it is possible that the wind tunnel may have had lower or higher $T_u$. A lower $T_u$ (i.e. $< 0.1\%$) is typical of modern low turbulence wind tunnels and would have the effect of shifting the predicted transition points downstream and the drag polars downwards (to lower drag values).

### 4 Conclusions

A RANS flow solver in two dimensions has been extended to incorporate an iterative laminar-turbulent transition prediction framework. The proposed frame-
The work presents a good compromise between accuracy, efficiency, and robustness that is well suited for aerodynamic shape optimization. The predictive capabilities of the framework show good agreement with numerical and experimental airfoil transition data. Future work will aim to incorporate the transition prediction capability into an adjoint-based aerodynamic shape optimization framework.

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REFERENCES


APPENDIX

In the new compressible form of the AHD criterion [24], the constants A through F are computed as a function of the freestream Mach number, $M_{\infty}$, as follows:

\[
\begin{align*}
A &= 98.64M_{\infty}^5 - 356.44M_{\infty}^2 + 117.13M_{\infty} - 236.69 \\
B &= -13.04M_{\infty}^4 + 38.5M_{\infty}^3 - 30.07M_{\infty}^2 + 10.89M_{\infty} + 22.7 \\
C &= 0.21M_{\infty}^3 + 4.79M_{\infty}^2 - 1.76M_{\infty} + 22.56 \\
D &= -3.48M_{\infty}^3 + 6.26M_{\infty}^2 - 3.45M_{\infty}^2 + 0.23M_{\infty} + 12 \\
E &= 0.6711M_{\infty}^3 - 0.7379M_{\infty}^2 + 0.167M_{\infty} + 51.904 \\
F &= 0.03016M_{\infty}^5 - 0.7061M_{\infty}^4 + 0.3232M_{\infty}^3 \\
&
-0.0083M_{\infty}^2 - 0.1745M_{\infty} + 14.6
\end{align*}
\]